Bachelor of Science (B.Sc.) Semester—III Examination

MATHEMATICS

(M6–Differential Equations and Group Homomorphism)

Paper—VI

Time—Three Hours] [Maximum Marks—60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. 1 to 4 have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Prove that :

(i) $J'_n(x) = J_{n-1}(x) - \frac{n}{x} J_n(x)$

(ii) $J'_n(x) = \frac{n}{x} J_n(x) - J_{n+1}(x)$

(iii) $2J'_n(x) = J_{n-1}(x) - J_{n+1}(x)$

(iv) $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$. 6

(B) Prove that :

(i) $J_{-3/2}(x) = \sqrt{\left(\frac{2}{\pi x}\right)}\left[-\frac{\cos x}{x} - \sin x\right]$

(ii) $J_{3/2}(x) = \sqrt{\left(\frac{2}{\pi x}\right)}\left[\frac{\sin x}{x} - \cos x\right]$ 6

OR

(C) Prove that $n p_n = x p_n' - p_{n-1}'$. 6

(D) Prove that $\int_{-1}^{1} x p_n(x) p_{n-1}(x)\, dx = \frac{2n}{4n^2 - 1}$. 6

UNIT—II

2. (A) If L(f(t)) = F(s), then prove that :

$$L\left[t^n f(t)\right] = (-1)^n \frac{d^n}{ds^n} F(s)$$

Hence evaluate L[t cos 2t]. 6

(B) If L(f(t)) = F(s), then prove that :

$$L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(u)\, du$$

provided $\lim_{t \to 0} \frac{f(t)}{t}$ exists. Hence prove that :

$$L\left(\frac{\sin t}{t}\right) = \tan^{-1}(1/s).$$ 6

OR

(C) Prove that :

$$L[f''(t)] = s^2 F(s) - s f(0) - f'(0),$$

where $F(s) = L(f(t))$ and $f(t), f'(t), f''(t)$ are continuous for $t > 0$. Hence find $L(\cos 2t)$. 6

(D) Evaluate $L^{-1}\left[\dfrac{1}{(s+1)(s^2+1)}\right]$ by using convolution property. 6

UNIT—III

3. (A) Solve $y'' + y' - 2y = t$, $y(0) = 1$, $y'(0) = 0$, where $y = y(t)$. 6

(B) Solve $x'' + 3x - 2y = 0$, $x'' + y'' - 3x + 5y = 0$ with $x(0) = x'(0) = y(0) = y'(0) = 0$, where $x = x(t)$, $y = y(t)$. 6

OR

(C) Find the Fourier cosine transform of $x^n e^{-ax}$. 6

(D) Solve $\dfrac{\partial U}{\partial t} = \dfrac{\partial^2 U}{\partial x^2}$, $0 < x < \infty$, $t > 0$ subject to $U(0, t) = 0$, $U(x, 0) = f(x)$, $U(x, t)$ is bounded. 6

UNIT—IV

4. (A) Prove that every quotient group G/H of a given group G is a homomorphic image of G. Also prove that the kernel of f is H. 6

(B) If G is the group of real numbers under addition and G′ = G, prove that $f : G \to G'$ defined by $f(x) = 12x, \forall x \in G$ is a homomorphism and one-to-one. 6

OR

(C) If H be a subgroup of a group G and N be a normal subgroup of G, then prove that $H \cap N$ is a normal subgroup of H. 6

(D) If N and M are two normal subgroups of a group G such that $N \cap M = \{e\}$, then show that every element of N commutes with every element of M. 6

UNIT—V

5. (A) Show that $J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 4^2} - \frac{x^6}{2^2 4^2 6^2} + \ldots$ 1½

(B) Prove that $x^2 = \frac{1}{3} P_0(x) + \frac{2}{3} P_2(x)$. 1½

(C) Prove that $L[\sin^2 4t] = \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2 + 64}\right]$. 1½

(D) Find Laplace transform of $t^2 \cos at$. 1½

(E) For $y''' + 2y'' - y' - 2y = 0$, $y(0) = y'(0) = 0$ and $y''(0) = 6$, find $L(y)$, where $y = y(t)$. 1½

(F) Find finite Fourier cosine transform of $f(x) = x$, $0 < x < 1$. 1½

(G) Express a multiplicative group $G = \{1, -1, i, -i\}$ as a cyclic group. How many are the generators ? 1½

(H) Let G be the additive group of reals and G′, the multiplicative group of all positive reals. If $f : G \to G'$ be defined by $f(x) = e^x$, $\forall x \in G$ is homomorphism, then find kernel of f. 1½